

LETTER

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Universally charging protocols for quantum batteries: A no-go theorem

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Abstract – The effectiveness of a quantum battery relies on a robust charging process, yet these are often sensitive to the initial state of the battery. We introduce the concept of a universally charging (UC) protocol, defined as one that either increases or maintains the average battery energy for all initial states, without ever decreasing it. We show that UC protocols are impossible for closed quantum batteries, thus necessitating interactions with auxiliary quantum systems. To that end, we prove a no-go theorem that prohibits UC protocols for closed quantum batteries with finite-dimensional Hilbert spaces. Leveraging a no-go theorem for topological quantum walks, we argue that even for infinite-dimensional Hilbert spaces, while unitary UC operators exist, they cannot be generated by physically reasonable Hamiltonians. However, regardless of the dimension, non-unitary UC protocols can be achieved in open quantum batteries. To illustrate this, we present a general model with a control qubit, whose state interpolates between universal-charging and universal-discharging protocols.

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Introduction. – Quantum technologies promise to outperform classical technologies by harnessing unique phenomena such as entanglement, superposition and coherence. Substantial evidence points to the plausibility of quantum advantage in computation [1,2], sensing [3,4], and cryptography [5]. Similarly, significant interest has also emerged in leveraging quantum correlations to enhance the performance of quantum thermodynamic devices, such as quantum heat engines and quantum batteries [6–8]. Early foundational work demonstrated that entanglement can significantly boost the charging power of quantum batteries [9].

Several studies have investigated the role of quantum coherence and entanglement in optimizing the charging process for quantum batteries [6,7,10–13]. Collective quantum effects can enhance the energy extraction process in ways that classical systems cannot replicate [14–16]. There are also various investigations of the dynamics of quantum batteries [10,17–20] that can lead to rapid energy redistribution and optimized charging processes [21]. The robustness of quantum batteries against environmental noise and decoherence have been used to infer bounds

on their operational performance [22–26]. In particular, it has been shown that global operations are required to obtain an advantage in charging speedups [27]. The experimental realization of quantum batteries is an active area of research. Some recent experimental demonstrations include platforms such as organic semiconductors in microcavities [28], nuclear spins [29], superconducting qutrits [30,31] and qubits [32].

Quantum battery performance is commonly evaluated using metrics such as charging power [10,18,22] and ergotropy —the maximum energy extractable via unitary operations [14,33–36]. When the battery’s initial state is known exactly, the maximally charging unitary protocol is the one that maximizes work extraction with respect to the negative battery Hamiltonian [33].

However, such protocols are highly state-dependent: a unitary that maximally charges one state may maximally *discharge* another. Furthermore, learning the full state of a quantum system is a prohibitively expensive task for large Hilbert spaces [37]. Therefore, the construction of charging protocols that are robust to deviations in the initial state is practically important. Taking inspiration from classical (electrochemical) batteries, which use fixed charging procedures regardless of initial state, we introduce the concept of “universal charging” (UC): protocols that never

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reduce—and ideally increase—the average battery energy for any initial state.

While one could always reset the battery to its ground state before charging, this incurs thermodynamic costs (for example, the energy cost of measurements, discussed in refs. [38,39]). Similarly, a charged battery typically loses energy over time and evolves into an unknown state due to environmental interactions when left idle [40]. In such cases, UC protocols can enable partial recovery of the lost energy without requiring state measurements, and reinitialization followed by explicit recharging.

We show that UC protocols can be achieved only for open quantum batteries, thus necessitating an auxiliary system or bath. To that end, we show that for a closed quantum battery, it is impossible to construct a UC protocol. In some sense, this is similar to the no-cloning theorem in quantum mechanics, which states that it is impossible to copy an unknown quantum state [41]. Furthermore, we show that for any given unitary protocol, the amount of energy charged, averaged across all possible initial states is identically zero. Therefore, a charging protocol that is optimized for a certain initial state, might be ineffective or may even discharge the battery when the initial state deviates significantly from the intended one¹.

This theorem is directly applicable only to quantum batteries with finite-dimensional Hilbert spaces. However, it can be partially overcome if the battery has an infinite-dimensional Hilbert space, or if the battery is an open quantum system, resulting in a non-unitary charging protocol. In the former case, we show that if the battery Hamiltonian has a single-sided infinite energy ladder (similar to the spectrum of a harmonic oscillator), unitary UC protocols still do not exist. However, UC unitaries, such as the energy-shift operator [43], do exist for double-sided infinite energy ladders. By interpreting the energy-shift operator as a quantum walk [44] in the energy space, we show that it has a non-zero topological index [45], and consequently, cannot be generated by any Hamiltonian that is local [46] in the energy basis of the battery Hamiltonian, and therefore is not physically realizable. We argue that this property is more general, and all UC unitary protocols are thus physically unrealizable on double-sided energy ladders.

We demonstrate the existence of *non-unitary* UC protocols by presenting a recipe for constructing them. Taking inspiration from models of topological quantum walks [47] and chiral Floquet topological insulators [48], we present a UC protocol for any finite-dimensional quantum battery by coupling it to an auxiliary qubit. In the limit of an infinite-dimensional quantum battery Hilbert space, the

auxiliary qubit becomes a catalyst, and the protocol becomes unitary, overcoming the topological obstruction via a mechanism elaborated below.

Setup. – We consider closed as well as open quantum batteries. Following standard convention [40], the energy of the battery is computed with respect to the battery Hamiltonian H_B , so that the (average) battery energy $E = \text{Tr}[H_B \rho_B]$, with ρ_B denoting the battery density matrix. For an open quantum battery, the total Hilbert space $\mathcal{H}_{\text{total}} = \mathcal{H}_B \otimes \mathcal{H}_E$, where B and E denote the battery and environment, respectively. We denote the battery density matrix by $\rho_B = \text{Tr}_E \rho_{\text{total}}$, which is the partial trace of the full density matrix.

A charging protocol is a mapping from the space of battery density matrices to itself. They can therefore be expressed as CPTP (completely positive trace-preserving) quantum channels [49]. The (average) amount of energy charged by a protocol $\mathcal{M}$ for an initial state ρ_B is $\Delta_{\mathcal{M}} E = \text{Tr}(H_B \{\mathcal{M}[\rho_B] - \rho_B\})$. We will drop the subscript $\mathcal{M}$ for ΔE when clear from the context. We consider cases where $\dim \mathcal{H}_B$ is finite as well as infinite, assuming a countable spectrum in the latter case.

We define a universally charging (UC) protocol $\mathcal{M}$, as one for which $\Delta_{\mathcal{M}} E(\rho_B) \geq 0 \forall \rho_B$ and $\Delta_{\mathcal{M}} E(\rho_B) > 0$ for at least one ρ_B . (The latter condition excludes trivial maps such as the identity map.) Universally discharging (UD) protocols can be defined similarly.

A UC map $\mathcal{M}$ is unitary if there exists a unitary operator U , such that $\mathcal{M}[\rho] = U \rho U^\dagger$ for any ρ . If U is a UC unitary operator, then its conjugate, $U^\dagger$, defines a UD unitary protocol.

Closed batteries with finite-dimensional Hilbert spaces. – Let us start by considering closed quantum batteries so that charging protocols are necessarily unitary. First, we note that for (almost) every density matrix, there exists a unitary map that increases its energy. This can be seen easily by noting that the problem of charging with respect to H_B is equivalent to discharging with respect to $-H_B$. Thus, for a state ρ , the unitary operation resulting in the largest charging ΔE is the one that corresponds to the ergotropy of ρ with respect to $-H_B$ [33]. If we are only allowed to implement from a fixed set of unitary operations, one can still find a maximizing protocol, as shown in the Supplementary Material [Supplementarymaterial.pdf](#) (SM).

Flipping the question around, we ask whether a protocol can charge every state, and find the answer to be negative: There does not exist any UC unitary protocol for a battery with a finite-dimensional Hilbert space. We call this the *no-charging theorem*.

Proof. Assume $\Delta E(\rho_S) \geq 0$ for all ρ_S . Let $H_B = \sum_n E_n |n\rangle\langle n|$ (non-degenerate eigenstates). Express $U = \sum_{m,n} c_{m,n} |m\rangle\langle n|$. Now we set $\rho_S = |N\rangle\langle N|$. From $\Delta E(|N\rangle\langle N|) \geq 0$, deduce $|c_{m,N}| = \delta_{m,N}$. Therefore, $U|N\rangle = e^{i\theta_N} |N\rangle$. We now apply the same argument inductively for $\rho_S = |N-1\rangle\langle N-1|, \dots, |1\rangle\langle 1|$. It follows that

¹Similar observations in specific cases have been made in the literature. For example, as noted in ref. [42], a charging interaction has to be switched off when the fully charged state is reached, to avoid spontaneous discharging. However, to our knowledge, the impossibility of UC protocols for closed quantum batteries has not been appreciated in its generality.

$U|j\rangle = e^{i\phi_j}|j\rangle$ for all j . Since $U|j\rangle = e^{i\phi_j}|j\rangle$, $[U, H_B] = 0$. Thus, $\Delta E(\rho_S) = 0$ for all ρ_S , implying that U is not a UC protocol $\square$.

We note that a unitary UC protocol exists iff a unitary UD protocol also exists, as we show again in the SM.

Thus, it follows that there does not exist a universally discharging protocol if $\dim \mathcal{H}_B$ is finite.

We will now present a second, more illuminating proof based on Haar averages, for the non-existence of UC and UD unitary protocols for finite-dimensional $\mathcal{H}_B$. To that end, we show that for any arbitrary unitary U , the average of ΔE over all unitary rotations of any ρ_S equals zero, as follows:

$$\overline{\Delta E} = \overline{\text{Tr}[H_B(U\{G_p G^\dagger\}U^\dagger - G\rho G^\dagger)]} \quad (1a)$$

$$= \frac{1}{\dim \mathcal{H}_B} \text{Tr}[H_B(U\{\text{Tr}\rho\}\mathbb{I}U^\dagger - \{\text{Tr}\rho\}\mathbb{I})] \quad (1b)$$

$$= \frac{1}{\dim \mathcal{H}_B} \text{Tr}[H_B(UU^\dagger - \mathbb{I})] = 0. \quad (1c)$$

Here, the overbar denotes a Haar-average over unitary matrices G (see footnote 2). Thus, if $\Delta E(\rho_1) > 0$ for some ρ_1 , then there necessarily exist a ρ_2 for which $\Delta E(\rho_2) < 0$ (and vice versa). Since eq. (1) is valid for all U as well as all ρ_S , it follows that there cannot exist any UC or UD unitary protocol. In fact, we proved a stronger statement, which says that the amount charged, averaged over all density matrices, is identically zero, for all unitary protocols³.

The no-charging theorem can be intuitively understood by noting that unitary protocols are essentially basis transformations. To that end, let us consider a pure initial state $|\psi\rangle = \sum_n c_n |n\rangle$, where $|n\rangle$ denotes the n -th energy eigenvector. The battery energy depends on the probability weights $|c_n|^2$ through the expression $E = \sum_n E_n |c_n|^2$. A unitary UC protocol is one that transforms any initial state by moving the probability weights higher up in energy. Since a unitary UC protocol necessarily has this effect for all initial states, repeated applications should result in a drift towards one or more high-energy “attractor” states in the space of all density matrices. Since this is impossible for unitary transformations, the impossibility of constructing UC or UD protocols using them follows at an intuitive level.

For a two-level system with $H = |1\rangle\langle 1| + 0|0\rangle\langle 0|$, the energy change $\Delta E(\rho)$ under an arbitrary unitary U , is illustrated over the Bloch sphere in fig. 1, which represents all possible quantum states. The calculation of ΔE for an arbitrary density matrix can be done analytically, which is provided in the SM. However, one can see from the figure that given a U , $\Delta E > 0$ for one hemisphere of the Bloch sphere and $\Delta E < 0$ for the other, with a dividing plane determined by the choice of U . The no-charging theorem ensures that the volumes of regions with $\Delta E > 0$ and

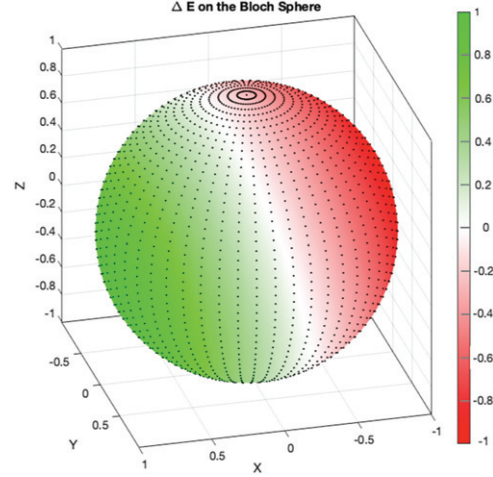


Fig. 1: $\Delta E(\rho)$ for a two-level system, with an arbitrary U , plotted over the Bloch sphere. For one hemisphere, $\Delta E < 0$, while for the other half, $\Delta E > 0$. The no-charging theorem implies that these two volumes compensate each other in general. Changing the unitary U will only rotate the plane dividing these regions.

$\Delta E < 0$ are equal, meaning energy gain in some states is exactly compensated by energy loss in others. Changing U rotates the dividing plane, altering the orientation of the hemispheres but maintaining the balance. This symmetry, visualized in fig. 1, highlights the constraints on energy transfer under unitary evolution.

Non-unitary universally charging protocols. –

The no-go theorem can be evaded if the protocol is non-unitary, therefore requiring interactions with the environment. Undesirable interactions with a heat bath are usually UD. Therefore, in theory, the existence of UC protocols seems plausible, if appropriately designed environmental interactions are intentionally introduced. Here, we present an explicit protocol (by specifying $H(t)$ for the battery-environment composite system) which is UC for any value of $\dim \mathcal{H}_B$, by coupling it to an auxiliary qubit. The protocol has some interesting and useful properties. First, the Hamiltonian $H(t)$ that generates the protocol is local in the energy basis of H_B , a requirement for physical Hamiltonians (see also discussion below regarding double-sided energy ladders). Second, the protocol can be tuned between the extreme limits of UC or UD by tuning the state of the control qubit. Finally, for the UC protocol, the value of ΔE is positive for every density matrix except for the highest-charged state (for which it is zero).

The protocol is specified by a Hamiltonian $H(t)$ that acts on $\mathcal{H}_{\text{total}} = \mathcal{H}_B \otimes \mathcal{H}_E$, with $\mathcal{H}_E$ denoting the auxiliary qubit Hilbert space. We denote the Pauli matrices by $\sigma_{x,y,z}$, while $\sigma_{\pm} := (\sigma_x \pm i\sigma_y)/2$. The eigenvectors of σ_z corresponding to eigenvalues $+1$ and -1 are denoted by $|\uparrow\rangle$ and $|\downarrow\rangle$, respectively. Let $\dim \mathcal{H}_B = N$. We consider an H_B with non-degenerate eigenvalues E_i , with eigenvectors $|i\rangle$ for $i = 1, \dots, N$, even though the model can

²We stress here that eq. (1) is applicable only if $\dim \mathcal{H}_B$ is finite.

³While this equation also appeared in ref. [50], its implications were not fully appreciated therein.

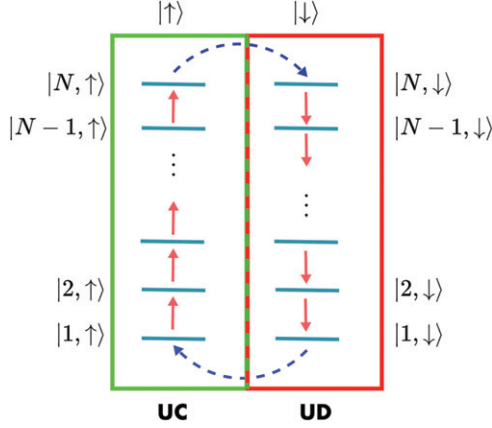


Fig. 2: Pictorial representation of the unitary operator generated by the protocol (3). When the control qubit is set to $|\uparrow\rangle$ ($|\downarrow\rangle$), the protocol is UC (UD).

be modified for the degenerate case. We define a “shift operator” s , similar to the truncated version of the annihilation ladder operator for simple harmonic oscillators, acting on the basis states as

$$s|n\rangle = \begin{cases} |n-1\rangle, & \text{if } n \in \{2, \dots, N\}, \\ 0, & \text{if } n = 1. \end{cases} \quad (2)$$

The driving protocol $H(t)$ is defined for $t \in [0, 1]$, and is split into two time steps:

$$H(t) = \begin{cases} -\pi (\mathbb{I} \otimes \sigma_x), & 0 \leq t < \frac{1}{2}, \\ \pi (s^\dagger \otimes \sigma_+ + s \otimes \sigma_-), & \frac{1}{2} \leq t \leq 1. \end{cases} \quad (3)$$

The corresponding time evolution operator that maps from $t = 0$ to $t = 1$, is given by $U = \mathcal{T} \exp(-i \int_0^1 H(t) dt)$, where $\mathcal{T}$ denotes the time-ordering operation.

The action of U on the various basis states of $\mathcal{H}_{\text{total}}$ is shown in fig. 2. It is straightforward to see that when the control qubit is set to $|\uparrow\rangle$, the map for the battery states can be expressed in its Kraus form [49] as

$$\mathcal{M}_{|\uparrow\rangle} = s^\dagger \rho s + \Pi_N \rho \Pi_N, \quad (4)$$

where $\Pi_N = |N\rangle\langle N|$ is the projector onto the N -th eigenstate. (See the SM for a derivation.) ΔE corresponding to any state $\Pi_n = |n\rangle\langle n|$ is

$$\begin{aligned} \Delta E(\Pi_n) &= \text{Tr}[H_B(\mathcal{M}_{|\uparrow\rangle}[\Pi_n] - \Pi_n)] \\ &= \begin{cases} E_{n+1} - E_n, & \text{if } n \in \{1, \dots, N-1\} \\ 0, & \text{if } n = N \end{cases} \geq 0. \end{aligned} \quad (5)$$

It can be straightforwardly seen that this protocol is UC, and results in a positive energy change for any initial state of the battery, except for $\rho = \Pi_N$, for which the energy is unchanged. Furthermore, if the control qubit is set to $|\downarrow\rangle$,

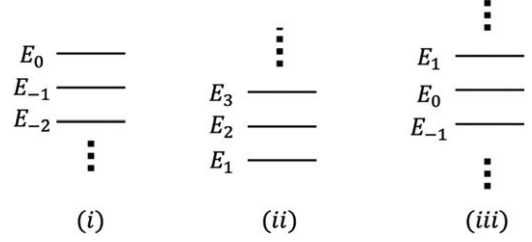


Fig. 3: The three cases for quantum batteries with infinite dimensional Hamiltonians that have a countable number of energies (and a finite degeneracy for each energy).

the protocol is UD, and the corresponding map assumes the form

$$\mathcal{M}_{|\downarrow\rangle} = s \rho s^\dagger + \Pi_1 \rho \Pi_1. \quad (6)$$

Interpolating the control qubit between $|\uparrow\rangle$ and $|\downarrow\rangle$ interpolates the protocol between these two limits. Unlike unitary protocols, for which, $\overline{\Delta E} = 0$ as shown previously, for general CPTP maps, $\overline{\Delta E} \neq 0$. Indeed, for $\mathcal{M}_{|\uparrow\rangle}$, we find that $\overline{\Delta E} = (E_N - E_1)/\dim \mathcal{H}_B > 0$ (see the SM, for a derivation).

Batteries with infinite-dimensional Hilbert spaces. – The two proofs for the no-charging theorem presented above are inapplicable when $\dim \mathcal{H}_B$ is infinite. We now consider situations where H_B has discrete and countably infinite energy levels. Again, we assume that each energy level is non-degenerate, although our arguments can be easily extended to the cases with finite degeneracies.

With these constraints, only three distinct cases arise: i) H_B with an energy ladder $E_0 > E_{-1} > \dots$, ii) H_B with an energy ladder $E_1 < E_2 < E_3 \dots$, and iii), a double-sided energy ladder with energies $\dots E_{-1} < E_0 < E_1 \dots$ (see fig. 3). Case i) has a finite maximum energy, but no minimum energy. Case ii) has a finite minimum energy, but no maximum energy, while case iii) corresponds to the situation with neither a minimum energy, nor a maximum energy. As an example, note that the Hamiltonian of a simple harmonic oscillator belongs to category ii), while its negative belongs to category i). Furthermore, in all of these cases, the energy levels are not required to be equally spaced. We will show that there are no UC or UD unitary protocols for the first two cases, while for the third UC and UD unitary protocols do exist. For all cases, we denote the eigenvector corresponding to E_n by $|n\rangle$.

Let us consider the first case, in which there is a finite largest energy E_0 . The impossibility of a UC protocol can be proven similarly to the case of finite-dimensional $\mathcal{H}_B$. Specifically, we can show that if U is a UC unitary protocol, then $U|k\rangle = e^{i\phi_k}|k\rangle$ for all k , and consequently, $[U, H_B] = 0$. This implies that $\Delta E = 0$ for all ρ_B , contradicting the assumption.

The second case is similar to the spectrum of a simple harmonic oscillator, but without requiring the energies to

be equally spaced. The impossibility of a UC unitary protocol for this case follows immediately from the first case. Let us assume that U is a UC protocol with respect to H_B of type ii). For any system, the Hermitian conjugate of a UC protocol is a UD protocol (see the SM for a proof). Hence, $V = U^\dagger$ must be a UD protocol, so that $\text{Tr}[H_B(V^\dagger \rho_B V - \rho_B)] \leq 0 \forall \rho_B$. This implies that $\text{Tr}[H'_B(V^\dagger \rho_B V - \rho_B)] \geq 0 \forall \rho_B$, where $H'_B = -H_B$ is a Hamiltonian of type i). Using this mapping, it can be seen that a UC unitary protocol for type ii) exists iff one also exists for type i). It follows that there are no UC unitary protocols for case ii). The impossibility of unitary UD protocols also follows similarly for the two cases.

In contrast, we note that unitary UC protocols *do exist* for case iii), which has a double-sided energy ladder. The simplest example of a unitary UC protocol is one corresponding to the energy translation operator

$$T = s^\dagger = \sum_{j=-\infty}^{\infty} |j+1\rangle\langle j|, \quad (7)$$

which is simply the conjugate of the double-sided, untruncated version of eq. (2). In fact, T corresponds to a *strictly* universally charging protocol, since $\Delta E = \text{Tr}[H_B(s^\dagger \rho s - \rho)]$ is strictly positive for all initial density matrices. Let us note that similar energy-shift operators have been employed in proposals of catalytic coherence [43] where it is implemented in an auxiliary “weight” system [51–53].

While any unitary operation can be defined mathematically without reference to a Hamiltonian, practical implementations (such as quantum gates) in quantum mechanics always occur via the time-evolution operator, which is generated by the Hamiltonian of the system. Although T is a unitary UC protocol, we will now show that it cannot be implemented physically, under reasonable assumptions. To that end, we will show that T cannot be “locally generated”, *i.e.*, generated by a Hamiltonian $H(t)$ that is local (see condition eq. (8b)) in the eigenbasis of H_B , if the battery is closed. We recall that a unitary operator U is said to be locally generated [46] if there exists some $H(t)$ for $0 < t < 1$ such that

$$U = \mathcal{T} \exp \left(-i \int_0^1 H(t) dt \right) \quad (8a)$$

$$\text{with } \langle n | H(t) | n' \rangle \leq C e^{-|n-n'|/l}, \quad (8b)$$

for all integers n, n' and some constants l, C . We expect the locality condition eq. (8b) to be a requirement for all “physical” Hamiltonians⁴.

Unitary operators such as T from eq. (7) can be interpreted as “one-dimensional quantum walks” operating in

⁴The double-sided energy ladder itself is often considered to be unphysical, since it lacks a ground state. However, we show that even if such a spectrum were assumed to exist, a UC protocol can still not be physically realized.

the energy space of H_B . An integer-valued topological invariant ν , called the “flow index” [45,54], characterizes 1d quantum walks. In our context, the flow of a unitary U can be defined as

$$\nu(U) = \sum_{x \geq 0 > y} [U_{xy}^* U_{xy} - U_{yx}^* U_{yx}], \quad (9)$$

where $U_{xy} := \langle y | U | x \rangle$, and $\circ^*$ denotes complex conjugation⁵.

Importantly, unitary operators with $\nu(U) \neq 0$ cannot be generated locally [45], *i.e.*, no $H(t)$ satisfying eq. (8) exists. It can be verified straightforwardly that the flow index of T is 1. It therefore follows that T cannot be generated locally. We also note that the flow index captures the net amount of upward shifting in energy of the weight of a density matrix. Consequently, we expect that unitary UC protocols that are more general than T should also have a non-zero flow index. It then follows that such protocols cannot be physically realized⁶.

However, a catalyst can help overcome this topological obstruction, as we now illustrate with an example. To this end, we consider the auxiliary qubit model $H(t)$ from eq. (3) for the double-sided ladder, but with the shift operator defined according to eq. (7). It is straightforward to show that the control-qubit-dependent maps (cf. eqs. (4)–(6)) on the battery are

$$\mathcal{M}_{|\uparrow\rangle} = s^\dagger \rho s, \quad (10a)$$

$$\mathcal{M}_{|\downarrow\rangle} = s \rho s^\dagger. \quad (10b)$$

Since these maps are unitary, the qubit is a catalyst. $\mathcal{M}_{|\uparrow\rangle}$ is a UC *unitary* protocol corresponding to the energy-shift operator T . Clearly, $H(t)$ is local (in fact strictly local) in the eigenbasis of H_B , in apparent contradiction to the topological obstruction. However, we note that the unitary operator generated by $H(t)$ over the full Hilbert space $\mathcal{H}_{\text{total}}$ has a flow index of zero. This can be be pictorially represented similarly to fig. 2. Overall, there is no *net* shift of energy in $\mathcal{H}_{\text{total}}$. Consequently, there is no topological obstruction.

Conclusions. – Fast charging protocols for quantum batteries are desirable for achieving higher efficiency. However, charging protocols are usually designed with certain initial states in mind, and are sensitive to deviations in the initial states. Such variations can be expected to arise due to undesirable environmental interactions and imperfect state preparation. In this letter, we introduced the concept of a universally charging (UC) protocol for quantum batteries. These protocols are defined by their

⁵For the case of degeneracies, the former is replaced by matrices and the latter is replaced by Hermitian conjugation, and a trace operation is introduced.

⁶Let us note that in the case of a continuum spectrum for the battery Hamiltonian, such as with $H_B = \int dE |E\rangle\langle E|$, there is no topological obstruction. The “momentum-like” operator $H = -i \frac{\partial}{\partial E}$ is local in the energy eigenbasis of H_B , and generates the translation operator $T(\Delta E) = \int dE |E + \Delta E\rangle\langle E|$ when applied for a time ΔE .

property of increasing or maintaining the average battery energy for all initial states, without ever decreasing it. UC protocols thus exemplify charging protocols robust to initial state variations.

We proved a no-go theorem that highlights a fundamental limitation in the design of such universal protocols for quantum batteries. Specifically, we showed that for closed quantum batteries with finite dimensional Hilbert spaces, it is impossible to design UC protocols. Similarly to the case of the no-cloning theorem, the ability to universally charge (or discharge) a quantum battery without observation or the use of auxiliary degrees of freedom is therefore restricted. The implications of this theorem are profound for the development of quantum energy storage technologies, and they suggest that state observation is important for a closed quantum battery. Observations, or partial observations, in the energy eigenbasis of the battery Hamiltonian, might destroy coherence, and thus limit the effective advantage obtained by quantum correlations.

Furthermore, we showed that the amount of energy charged averaged over all initial states is identically zero, highlighting the fragility of charging options available for closed quantum batteries. While UC protocols do exist for closed quantum batteries with double-sided infinite energy ladders, we argued that a topological obstruction makes these protocols unachievable via physical processes. Nonetheless, the no-go theorem as well as the topological obstruction can be overcome for open quantum batteries, for which we presented an explicit recipe for constructing UC protocols.

Various research directions are suggested by our work. We presented a particular Hamiltonian protocol that is universally charging with access to an auxiliary qubit. The construction of more general, robust protocols that are coupled to a thermal bath would be interesting. It would be useful to define and understand more general measures of robustness of charging protocols. The importance of open quantum system effects for designing UC protocols suggests that these might also be important while considering other measures of robustness.

We have shown that UC protocols inherently rely on open-system dynamics. Since these effects are typically considered detrimental in the context of other performance metrics—such as ergotropy and charging power—investigating the trade-offs between these competing aspects presents an intriguing avenue for future work. While we focused on protocols that charge every possible initial state, it may be more practical to design robust protocols tailored for situations where the battery’s exact state is unknown, but some partial information about it is still available. It would be interesting to prove our conjecture that UC unitary operators have a non-zero topological index (for the case of an infinite double-sided energy ladder), since it has direct implications on the physicality of such protocols. Understanding the trade-offs between robustness to deviations in initial states, and the speed of charging with optimized charging protocols would be

a practical and important research direction. Another promising direction is the development of UC protocols tailored to promising experimental platforms for quantum batteries.

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